Next Step: Theorem 1.6 (= Theorem 4.1)

Goal:

> Let (X, D) be a proper log smooth algebraic space and let
> $$\mathbb{V} = (V_{\mathbb{Z}}, F^{\bullet}V_{\mathcal{O}})$$
> be a polarizable integral pure Calabi–Yau variation of Hodge structures on $X \setminus D$ with unipotent local monodromy. Let M_X be the Hodge bundle on X. If M_X is integrable and has torsion combinatorial monodromy, then M_X is semiample.

It seems the line of the argument is as follows:

integrability + torsion combinatorial monodromy	(hypotheses of Theorem 4.1)
$\Longrightarrow R_{\text{curve}}$ is a closed constructible equivalence relation	(Corollary 3.10)
$\Longrightarrow X/R_{\text{curve}}$ exists as a definable topological quotient	(Remark 3.2)
$\Longrightarrow X/R_{\text{curve}}$ is algebraic	(Theorem 4.4)
$\Longrightarrow M_X^{\otimes k}$ descends to an ample bundle	(Lemma 3.16)
$\Longrightarrow M_X$ is semiample	(Theorem 4.1).

Talk 1. Integrability.

State Theorem 1.6/4.1 and define the notion of integrability (Definition 2.17). For this, you might need to introduce some of the notation from the beginning of subsection 2.5. Give a full proof of Lemma 2.18 if possible and do everything until including Lemma 2.21.

Talk 2. Torsion combinatorial monodromy

Do Section 2.8 and prove Lemma 2.22. Then define the notion of torsion combinatorial monodromy (Definition 2.23) and prove the GGR result (Theorem 2.24) and Lemma 2.25.

Now you can prove Corollary 4.2.

Talk 3. Equivalence relations I

Do the basics of equivalence relations on definable topological spaces (constructible, closed, Section 3.1).

Define R_{curve}: two points are equivalent if they can be connected by a proper connected curve on which M_X has degree zero (Section 3.2).

Compare R_{curve} with the Hodge-theoretic relation coming from the transcendental part of the variation (Section 3.2).

Explain why torsion combinatorial monodromy gives finite monodromy for the isotrivial transcendental variation along M_X-degree-zero curves (Lemma 3.3).

Talk 4. Equivalence relations II

Describe the stratification of X induced by D and the limiting variations of mixed Hodge structures on the strata (Section 2.5). Define the CY-minimal quotient (Section 2.6).

Discuss the boundary conditions $(B1)$–$(B3)$ and why one is allowed to replace (X, D) by a suitable modification satisfying them (Section 3.3, especially Lemma 3.8).

Talk 5. The proof of Theorem 4.1.

Prove Theorem 4.4 and hence Theorem 4.1.

For this you might need to summarize the remainder of Section 3. (Or do we want to put this in yet another talk on equivalence relations?)